

* Atomicity of Gases :-

(ii)

(i) Mono-atomic gas :- A monoatomic gas molecule has one atom. Each molecule has three degrees of freedom due to translatory motion only.

Energy associated with each degree of freedom $= \frac{1}{2} kT$

Energy associated with three degree of freedom $= \frac{3}{2} kT$.

Consider one gram mole of gas.

Energy associated with one gram molecule of a gas $= N \times \frac{3}{2} kT$

$$= \frac{3}{2} (N \times k) T$$

$\because N \times k = R$

$$\therefore U = \frac{3}{2} RT$$

This energy of the gas is due to energy of its molecules. It is called internal energy U . For an ideal gas, it depends upon temperature only.

$$\therefore C_v = \frac{dU}{dT} = \frac{3}{2} R$$

But, $C_p - C_v = R$

$$\therefore C_p = C_v + R$$

$$= \frac{3}{2} R + R = \frac{5}{2} R$$

For a mono atomic gas, $\gamma = \frac{C_p}{C_v}$

$$\therefore \gamma = \frac{\frac{5}{2} R}{\frac{3}{2} R} = \frac{5}{2} \times \frac{2}{3} = 1.67$$

The value of γ is found to be true experimentally for mono atomic gases like argon and helium.

(ii) Diatomic gas:— A diatomic gas molecule has two atoms. Such a molecule has three degrees of freedom of translation and two degrees of freedom of rotation.

Energy associated with each degree of freedom
= $\frac{1}{2} kT$

Energy associated with 5 degrees of freedom = $\frac{5}{2} kT$
consider one gram molecule of a gas.

Energy associated with 1 gm molecule of diatomic

$$\text{gas} = N \times \frac{5}{2} kT = \frac{5}{2} RT \quad \because N \times k = R$$

$$\therefore U = \frac{5}{2} RT$$

$$C_v = \frac{dU}{dT} = \frac{5}{2} R$$

But $C_p - C_v = R$

$$\therefore C_p = C_v + R = \frac{5}{2} R + R = \frac{7}{2} R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{2} \times \frac{2}{5} = \frac{7}{5} = 1.40$$

The value of $\gamma = 1.40$ has been found to be experimentally for diatomic gases like H_2 , O_2 , N_2 , etc.

ii) Triatomic gas: —

(a) A triatomic gas having 6 degrees of freedom has an energy associated with one ^{gm}molecule

$$= N \times \frac{6}{2} kT = 3RT$$

$$U = 3RT$$

$$C_v = \frac{dU}{dT} = 3R$$

$$\text{But, } C_p - C_v = R$$

$$C_p = C_v + R = 3R + R = 4R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{4R}{3R} = 1.33$$

) A triatomic gas having 7 degrees of freedom has an energy associated with one gm molecule = $N \times \frac{7}{2} kT = \frac{7}{2} R$

$$U = \frac{7}{2} R$$

$$C_v = \frac{dU}{dT} = \frac{7}{2} R$$

$$\text{But } C_p - C_v = R$$

$$C_p = C_v + R$$

$$= \frac{7}{2} R + R = \frac{9}{2} R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{\frac{9}{2} R}{\frac{7}{2} R} = \frac{9}{2} \times \frac{2}{7} = \frac{9}{7} = 1.28$$

Thus the value of γ , C_p and C_v can be calculated depending upon the degrees of freedom of a gas molecule.